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FWF
Der Wissenschaftsfonds.

UNIVERSITÄT KLAGENFURT

Modeling - Analysis - Optimization
DOC.FUNDS DOCTORAL SCHOOL

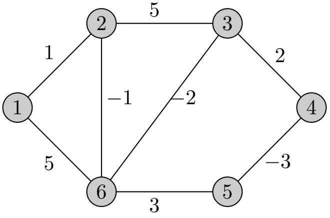
Variable Fixing for Max-Cut

Overview

1. The Max-Cut Problem
2. Reduced Cost Fixing in Linear Programming
3. Variable Fixing for Semidefinite Programming

The (Weighted) Max-Cut Problem

Given: undirected graph $G = (V, E)$ with edge weights $w \in \mathbb{R}^E$



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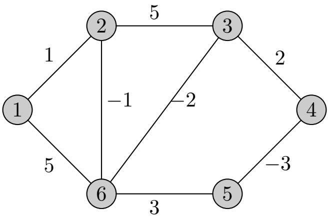

Goal: find a maximum cut in G, i.e., an optimal solution of

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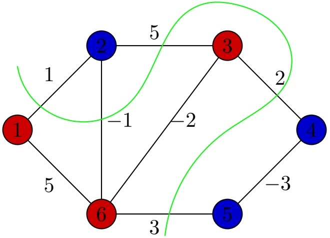

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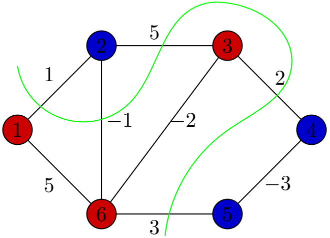

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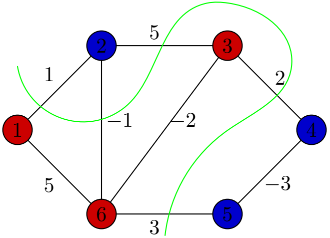

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- (MC) is $\mathcal{NP}$-hard
- for $C = \frac{1}{4} L(G)$, (MC) is a special case of

$$\begin{aligned} \max \quad & x^\top C x \\ \text{s.t.} \quad & x \in \{-1, 1\}^n \end{aligned}$$

Reduced Cost Fixing in Linear Programming

$$\max \ x^\top C x \quad \text{s.t.} \quad x \in \{0,1\}^n \tag{QP}$$

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- $b^\top \hat{u} - \hat{u}_{ij} \leq \bar{c} \quad \Rightarrow \quad$ we can fix $y_{ij} = 1$!

Variable Fixing for Semidefinite Max-Cut Relaxations

$$\max \; x^\top C x \quad \text{s.t.} \quad x \in \{-1,1\}^n \tag{MC}$$

By introducing $X = xx^\top$, we have $x^\top C x = \langle C, xx^\top \rangle = \langle C, X \rangle$, where $\langle A, B \rangle := \operatorname{tr}(B^\top A)$.

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$\Rightarrow$ dual variables have to be computed/constructed if needed

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Easy case: $e^\top \hat{u} + b_0 u_0 \leq \bar{c}$ and $\hat{Z} + u_0 A_0 \succeq 0$ for some $u_0 \in \mathbb{R}$

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  - gradient-based algorithm
  - line search in some parameter

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